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13. ABSTRACT (Maximum 200 words) Over the period of research for this grant the accomplishments were research on: (1) Tail behavior of certain probability laws, and the behavior of Gaussian laws of translated balls. (2) Supports and densities of certain infinitely divisible (i.d.) laws and the zero-one dichotomy for certain non-linear functionals of i.d. random vectors. (3) The best linear predictors for certain i.d. processes.					
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FINAL REPORT ON THE RESEARCH ACCOMPLISHED
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RESEARCH REPORTS ACCOMPLISHED
DURING THE PERIOD AUGUST 91- MAY 93
UNDER THE RESEARCH GRANT # AFOSR 90-0168

1. Tomasz Byczkowski and Balram S. Rajput, "Tail behavior for semigroups of measures on groups," J. of Theoretical Probability, 5(1992), 537-544.
2. Jan Rosinski and Gennady Samorodnitsky, "Distributions of subadditive functionals of sample paths of infinitely divisible processes." Annals of Probability, 21(1993), 996-1014.
3. Jan Rosinski, "Remarks on strong exponential integrability of vector valued random series and triangular arrays." To appear in Annals of Probability.
4. Werner Linde and Jan Rosinski, "Exact behavior of Gaussian measures of translated balls in Hilbert spaces." To appear in J. of Multivariate Analysis.
5. Balram S. Rajput, "Supports of certain infinitely divisible probability measures on locally convex spaces". Annals of Probability, 21(1993), 886-897.
6. Balram Rajput, K. Rama-Murthy and T. Żak, "Supports of semi-stable probability measures on LCS, submitted to J. of Multivariate Analysis.
7. Tomasz Byczkowski, John P. Nolan and Balram Rajput, "Approximation of multidimensional stable densities". To appear in J. of Multivariate Analysis.
8. John Nolan and Balram Rajput, "Numerical Calculation of Multidimensional stable densities", submitted to Computational Statistics.
9. Jan Rosinski, Gennady Samorodnitsky and Murad S. Taqqu, "Zero-one laws for multilinear forms in Gaussian and other infinitely divisible random variables". To appear in J. of Multivariate Analysis.
10. Balram S. Rajput and Carl Sundberg, "On some extremal problems in H^p and the prediction of L^p -representable stochastic process, $1 \leq p < \infty$ ". To appear in Probability Theory and Related Fields.
11. Jan Rosinski, Stamatis Cambanis, V. Mandrekar, and Donatas Surgailis, "Stable generalized moving averages." To appear in Probability Theory and Related Fields.
12. Jan Rosinski "On Uniqueness of the Spectral Representation of Stable Processes," submitted to J. of Theoretical Probability.

We have made a significant achievement towards completing the research goals set out in our proposal. In addition to the research which has already been reported in our earlier reports, we have also completed several more research papers since August 91 under this proposal. These fall under the following four categories:

- I. Tail behavior of certain probability laws, and the behavior of Gaussian laws of translated balls.
- II. Supports and densities of certain infinitely divisible (i.d.) laws and the zero-one dichotomy for certain non-linear functionals of i.d. random vectors.
- III. The best linear predictors for certain i.d. processes.
- IV. Structural analysis of stable processes.

The accomplished research is reported in the twelve [1-12] papers. In the following, we summarize and highlight this research.

The research accomplished under Topic I is contained in [1-4]. A precise description of the tail behavior of a probability measure provides valuable information about its distribution. Important research in this area is presently being carried out by a number of researchers. The work [1] is devoted to study the tail behavior of convolution semigroups of probability measures defined on measurable groups. This work significantly extends and considerably simplifies earlier results in this direction which were proved under the restrictive setting on measurable vector spaces and for special classes of convolution semigroups of probability measures (e.g., semigroups of α -stable probability measures). As an application of the main result obtained in [1], we show that one can obtain tail behavior of the maximum of certain group-valued stochastic

processes.

Subadditive functionals on the space of sample paths include suprema, integrals of path, oscillation on sets, and many others. In paper [2], an optimal condition is obtained which ensures that the distribution of a subadditive functional of sample paths of an i. d. process belongs to the subexponential class of distributions. This result is achieved by establishing the exact tail behavior for the distribution of such functionals. The class of subexponential distributions has been studied in the past in many different areas such as branching processes, extreme values and reliability. Our results provide new applications for such distributions. Furthermore, applying these results to specific processes and subadditive functionals (suprema, for example), we get immediately many earlier known results on the tail behavior which were obtained in the past by entirely different methods.

The paper [3] refines the results of Talagrand, and Kwapien-Szulga on the strong exponential integrability of vector random series; the results obtained are best possible. This paper also provides an optimal condition on the integrability of the norm of an i. d. random vector with Lévy measure having bounded support. As an application of the last result, one can derive a sharp tail estimate of subadditive path-functionals of certain i. d. processes.

The problem considered in [4] can be described as follows. Given a statistic based on a quadratic form in a discrete time Gaussian process, one would like to know how much the distribution of such a statistic changes when the mean of the Gaussian process changes. This problem can also be viewed as a study of the power function for a certain statistical test. In [4] we discover that the change of the mean is equivalent to adding of

an independent infinitely divisible term to the original quadratic form in zero-mean Gaussian process. As an straightforward application of this fact we provide a simple and short proof of Zak's theorem and compute Gateaux derivative of the Gaussian probability of a translated ball. This gives an alternative way to establish an analogy of heat equation on infinite dimensional Hilbert spaces. Results of this paper also provide some surprising facts on the behavior of a Gaussian measure of translated discs on the plane.

The papers [5-9] consist of the accomplished research under Topic II. The work [5] is a revised and updated version of an earlier preliminary research report entitled "On the structure of the supports of i.d. probability measures on locally convex spaces". It is devoted to resolve two long standing conjectures concerning the structure of the supports of i.d. measures on linear spaces. This work also yields an affirmative answer to the question, open for some time, of whether the support of a α -stable probability measure, $1 \leq \alpha < 2$, on a infinite dimensional Banach space is a translate of a linear space. The paper [6] provides analogous results for semi-stable measures.

The knowledge of various properties of densities of i.d. probability laws on \mathbb{R}^d is very useful information both for theoretical and practical considerations. Yet, with the exception of certain very special cases, very little is known about the properties of probability densities of most i.d. laws on \mathbb{R}^d , $d \geq 2$. One of the main reasons for this is that these densities in two or more dimensions do not generally have explicit formulae. This is true in particular for α -stable densities; these stable densities are usually described in terms of their spectral measure. The main result of [7] shows that the density of any α -stable law on \mathbb{R}^d , $d \geq 2$, can be uniformly approximated by the density of a α -stable

law having a discrete spectral measure with a finite number of atoms. Further, we provide in this paper, a concrete formula for the number of atoms and their weights. A similar result is proved for uniform approximation of probabilities involving any α -stable law on \mathbb{R}^d , $d \geq 2$. Since these stable densities and the probabilities can be numerically computed, one can use this to obtain considerable information about densities (and probabilities) of stable laws with general spectral measures. In paper [8], we provide a program to calculate two dimensional stable densities that uses a recent two dimensional adaptive quadrature routine.

Zero-one laws for Gaussian and stable processes are basic to the study of sample paths because they state that, with the possible exception of a null set of paths, all sample paths share a given property or none of them do. The work [9] extends the scope of the zero-one laws for the processes represented by multiple Gaussian, multiple stable, and multiple type-G stochastic integrals. In this paper, we have developed a method similar to the so-called "decoupling technique" which can be implemented without any metric structure.

The research accomplished under Topic III is reported in [10]. In this paper, by using a new approach, we provide, under suitable conditions, solutions to two dual extremal problems in the upper half plane and in the disc. In turn, utilizing these results, we furnish a unified solution to the important linear prediction problems for a class of continuous as well as discrete parameters L^p -representable stochastic processes for all $1 \leq p < \infty$. Two of the four results presented here are new; and the method of proof of the rest of the results provide alternate new, much simpler and shorter proofs for some

earlier known results. This paper, in particular, provides the formulae for the best linear predictors for a class of L^p -representable processes, $1 \leq p < \infty$; these formulae when specialized to $p = 2$ coincide with the well known best linear Gaussian predictors due to Wiener and Kolmogorov. This paper encompasses the earlier works due to Cambanis and Soltani; and Rajput, Rama-Murthy and Sundberg where best linear predictors are derived for certain L^p -representable processes for $1 \leq p < 2$.

The research accomplished under Topic IV is reported in [11-12]. In [11], the class of (non-Gaussian) stable moving average processes is extended by introducing an appropriate joint randomization of the filter function and of the stable noise, leading to stable mixed moving averages. Their distribution determines a certain combination of the filter function and the mixing measure, leading to a generalization of a theorem of Kanter for usual moving averages. Stable mixed moving averages are ergodic and are not harmonizable. In [12], we proved that any two spectral representations of a symmetric stable process may differ only by a change of variable and a parameter-independent multiplier. Our result can immediately be used either to distinguish or to identify stable processes from various classes of interest. A characterization of stationary stable processes is also given.